NewHope, Frodo, in Between and Beyond

Instantiating and Implementing Lattice based Cryptography

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Outline

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Lattice Based Crypto

Theory:

- Strong asymptotic security guarantees
- Resistant to known quantum attack
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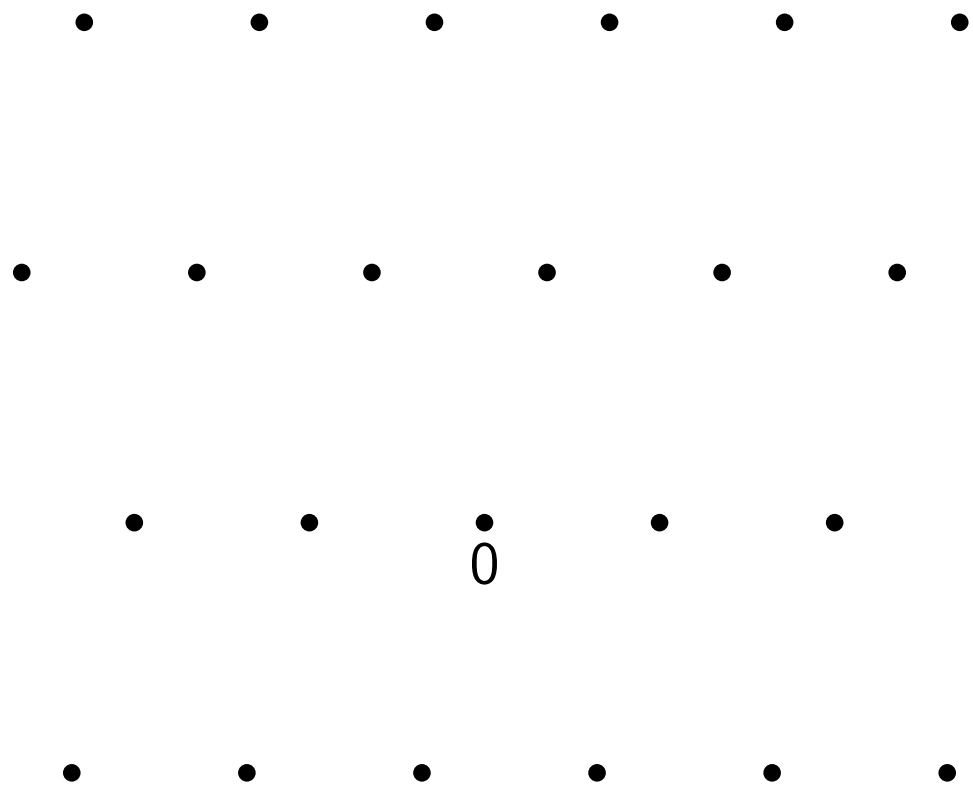
Practice/Deployment:

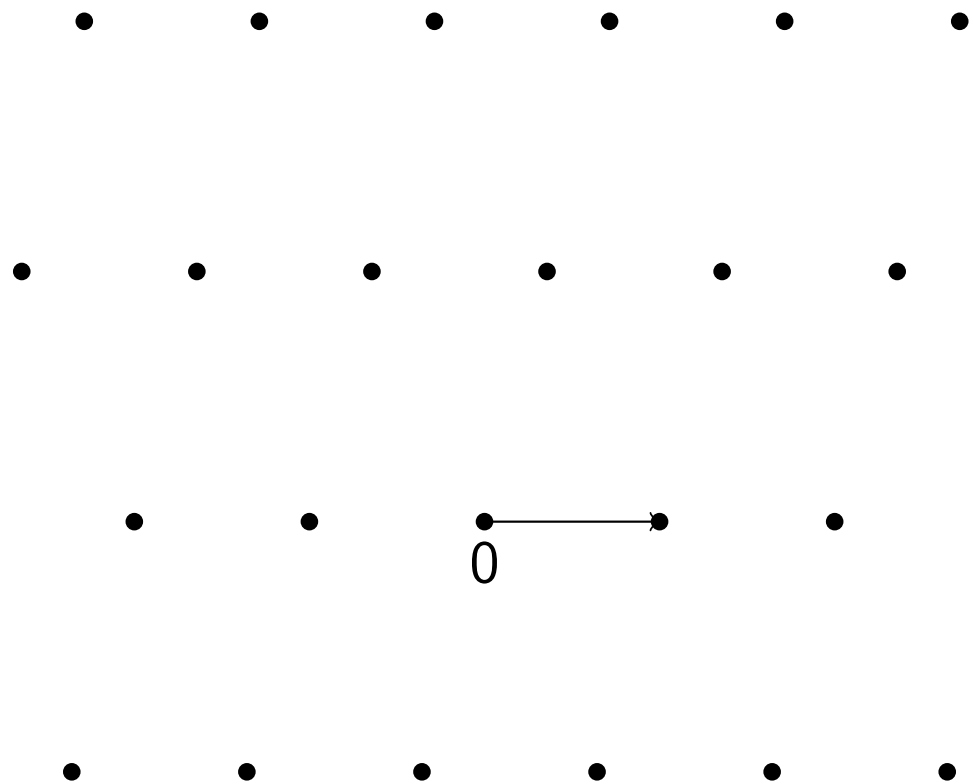
- NSA, NIST, CGS, recommend initiating transition to Post-Quantum crypto by 2020.
- Lattice based Crypto is (one of ?) the best candidate
- Far from ready for the real world

Lattices and hard problems



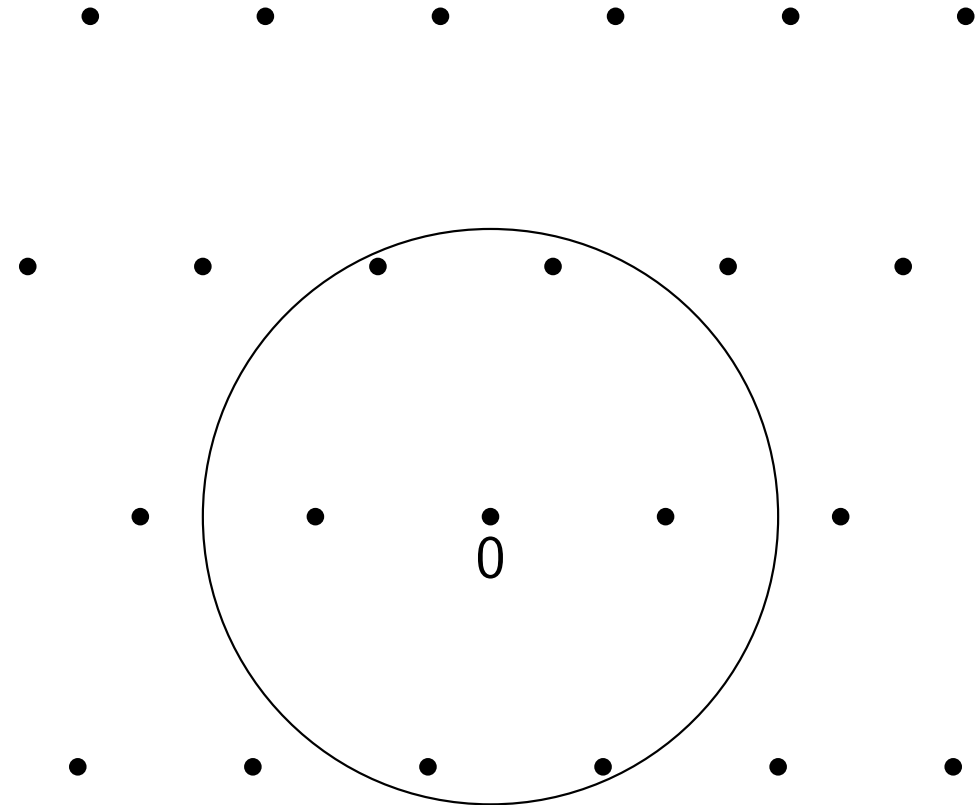
A lattice

Lattices and hard problems



The Shortest Vector Problem (SVP)

Lattices and hard problems



The Approximate Shortest Vector Problem (Approx-SVP)

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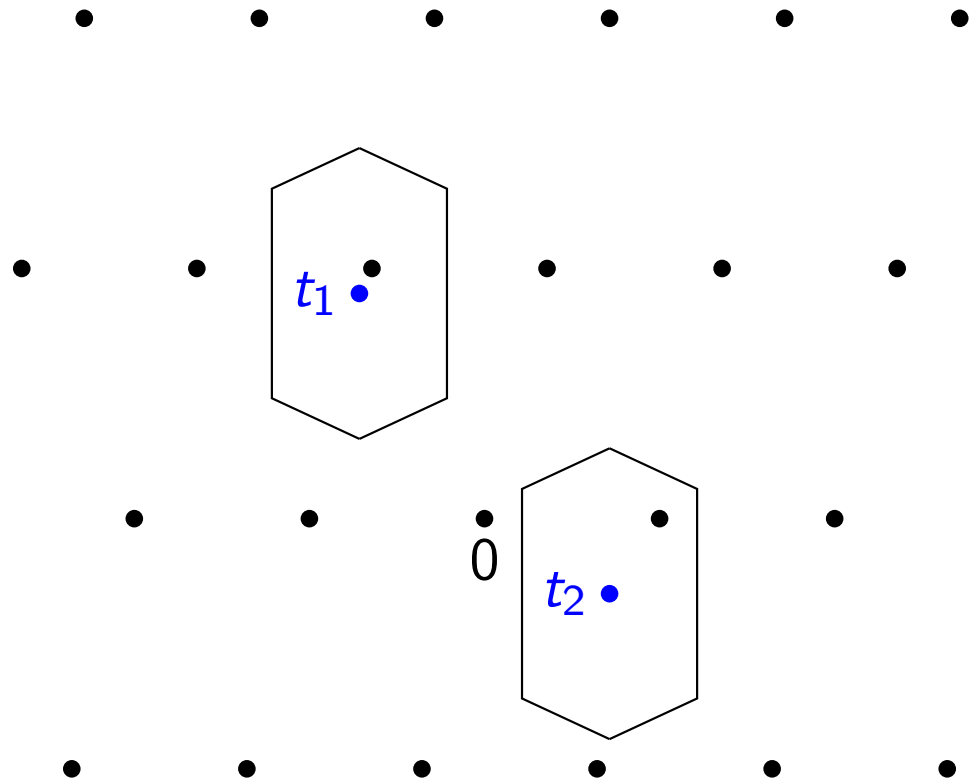

The Closest Vector Problem (CVP)

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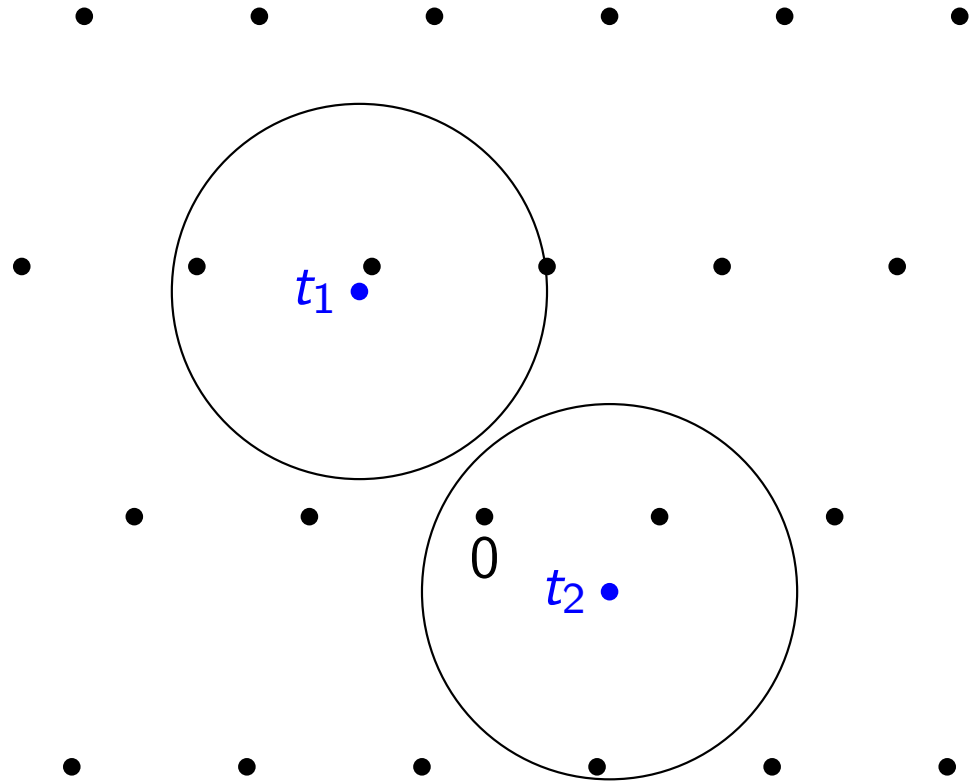

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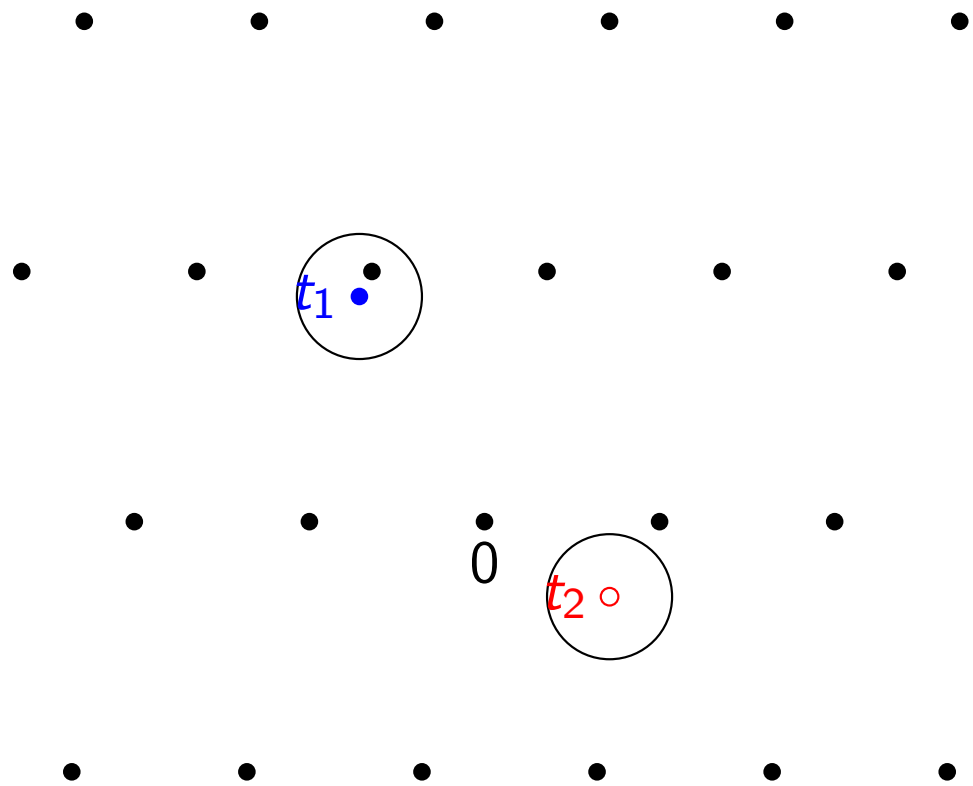


The Bounded Distance Decoding Problem (BDD)

Pre-Regev Lattice-based cryptography

Knowing a good basis (e.g. many approx-SVP solution) makes Approx-CVP and BDD easy

$\Rightarrow$ Public Key cryptography ?

BDD "based" encryption

pk A lattice L.

sk A good basis.

enc(m) View m as a lattice point $L(m)$.

$$c = L(m) + e$$

for some small e.

How to instantiate ?

What lattice ? What error distrib. ? Are messages random ?

Early approaches (NTRU / GGH) pretty heuristic [HPS98, GGH97]

Learning with Errors [BFKL94, Ale03, Reg05]

Definition (Learning with errors, Search and Decision)

Parameters: integers n, m, q, error distrib: ψ
Instance: $\mathbf{A} \leftarrow \mathcal{U}(\mathbb{Z}_q^{m \times n}), \mathbf{s} \leftarrow \mathbb{Z}_q^n, \mathbf{e} \leftarrow \psi^m$ (or $\mathbf{s} \leftarrow \psi^n$)
Given: $(\mathbf{A}, \mathbf{As} + \mathbf{e})$
Find: $\mathbf{s}$ / Distinguish: from uniform $\mathcal{U}(\mathbb{Z}_q^{m \times n+1})$

search-LWE as a BDD instance:

- $\mathbf{As}$ is a point in the lattice $L = \{\mathbf{Ax} + q\mathbf{y}, \mathbf{x} \in \mathbb{Z}^n, \mathbf{y} \in \mathbb{Z}^m\}$

Theorem (Quantum reduction, [Reg05])

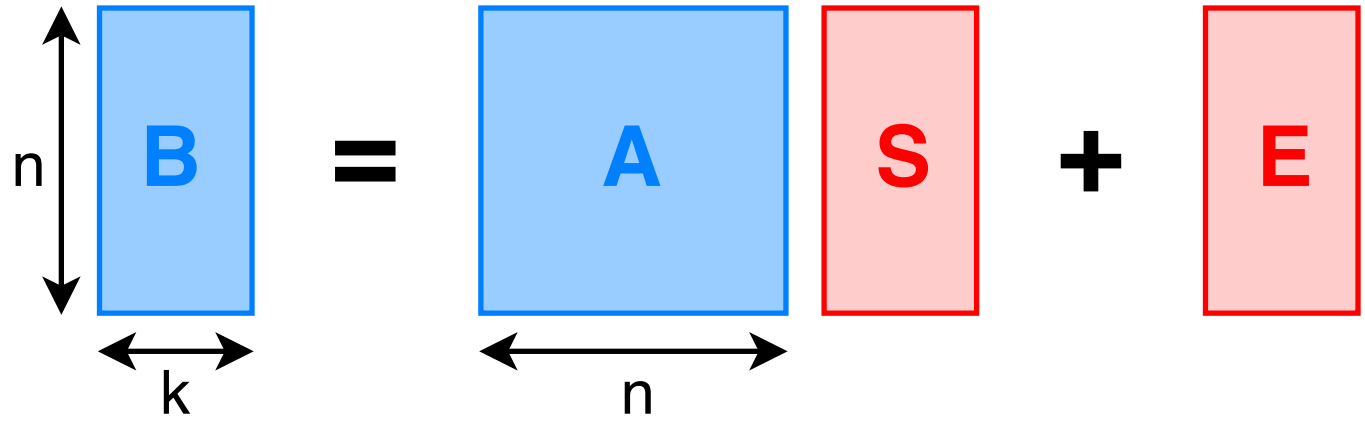
Search-LWE and Decision-LWE are at least as hard as Approx-SVP in the worst-case.

Note: Excellent theortical fundation, yet little practical meaning

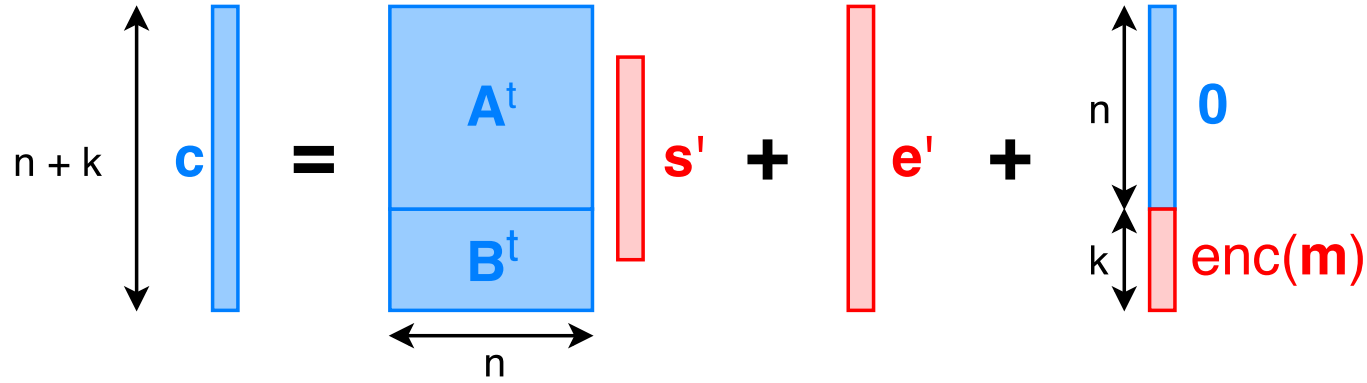
LWE-based encryption

Public parameter $\mathbf{A} \leftarrow \mathcal{U}(\mathbb{Z}_q^{n \times n})$

KeyGen()



$Encrypt_{\mathbf{b}}(m \in \{0,1\})$



The ring variant [LPR10, LPR13]

Replace matrices and vectors by elements of a (abelian) ring $\mathcal{R} = \mathbb{Z}[X]/(P(X))$.

Definition ($\mathcal{R}$-LWE (over simplified))

Parameters: integers n, m, q, error distrib: ψ
Instance: $a \leftarrow \mathcal{R}, s \leftarrow \mathcal{R}, e \leftarrow \psi$ (or $s \leftarrow \psi$)
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Theorem (Quantum reduction, [LPR10])

Search-$\mathcal{R}$-LWE are at least as hard as Ideal-Approx-SVP in the worst-case.
When $\mathcal{R}$ is cyclotomic ($\mathcal{R} = \mathbb{Z}[\omega_m]$), Decision-$\mathcal{R}$-LWE is at least as hard as Search-$\mathcal{R}$-LWE.

Outline

Ding et al. / Peikert's RLWE-based KEM

Parameters: q, n, χ

KEM.Setup() :

$\mathbf{a} \xleftarrow{\$} \mathcal{R}_q$

Alice (server)		Bob (client)
KEM.Gen($\mathbf{a}$) :		KEM.Encaps($\mathbf{a}, \mathbf{b}$) :
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$\bar{\mathbf{v}} \xleftarrow{\$} \mathrm{dbl}(\mathbf{v})$
KEM.Decaps($\mathbf{s}, (\mathbf{u}, \mathbf{v}')$) :	$\xleftarrow{\mathbf{u}, \mathbf{v}'}$	$\mathbf{v}' = \langle \bar{\mathbf{v}} \rangle_2$
$\mu \leftarrow \mathrm{rec}(2\mathbf{us}, \mathbf{v}')$		$\mu \leftarrow \lfloor \bar{\mathbf{v}} \rceil_2$

BCNS key exchange

- Bos, Costello, Naehrig, Stebila, IEEE S&P 2015:
  - Instantiate with concrete parameters
  - Integrate with OpenSSL $\rightarrow$ post-quantum TLS key exchange
  - Also: combined ECDH+RLWE key exchange

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- Parameters chosen by BCNS:
  - $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
  - $n = 1024$
  - $q = 2^{32} - 1$
  - $\chi =$ Gaussian of parameter σ
  - $\sigma = 8\sqrt{2\pi} \approx 3.192$
- Total communications: 2×4 KBytes

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- Total communications: 2×4 KBytes
- Claimed security level: 128 bits pre-quantum

A New Hope [Alkim, D., Poppelmann, Shwabe, 2016]

Awarded the **the Internet Defense Prize** (Usenix/Facebook)

Contributions:

- Improve failure analysis and error reconciliation
- Drastically reduce modulus q, **halving** communication
- Analysis of post-quantum security
- Use centered binomial noise ψ_k (reduction provided)
- Choose a fresh parameter **a** for every protocol run
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Closer to real-world cryptography:

- Implemented in Boring-SSL, tested in **Google Chrome**

Why sampling fresh parameter **a** at each run ?

We know how to build undetectable backdoors !

It seems simpler and safer to just avoid p.p. than to mount nothing-up-my-sleeve countermeasure.

Other downfall:

- All-for-the-price-of-one attack (e.g. logjam attack): one very large computation can break all instances
- Multi-targets attack becomes easier

Yet, very easy to avoid, at a reasonable cost.

Newhope: Security levels

Attack	samples m	BKZ b	Known Classical	Known Quantum	Best Plausible
BCNS proposal [BCNS15]: $q = 2^{32} - 1$, $n = 1024$, $\sigma = 3.192$					
Primal attack	1062	296	86	**77**	61
JarJar: $q = 12289$, $n = 512$, $\sigma = \sqrt{12}$					
Primal attack	623	449	131	**117**	93
NewHope: $q = 12289$, $n = 1024$, $\sigma = \sqrt{8}$					
Primal attack	1100	967	282	**253**	200

Note: This is according to a very pessimistic security analysis. In particular, this does not invalidate the security claim of [BCNS15].

Newhope: Efficiency

Communications:

$$\text{Alice} \xrightarrow{1824\text{Bytes}} \text{Bob}$$

$$\text{Alice} \xleftarrow{2048\text{Bytes}} \text{Bob}$$

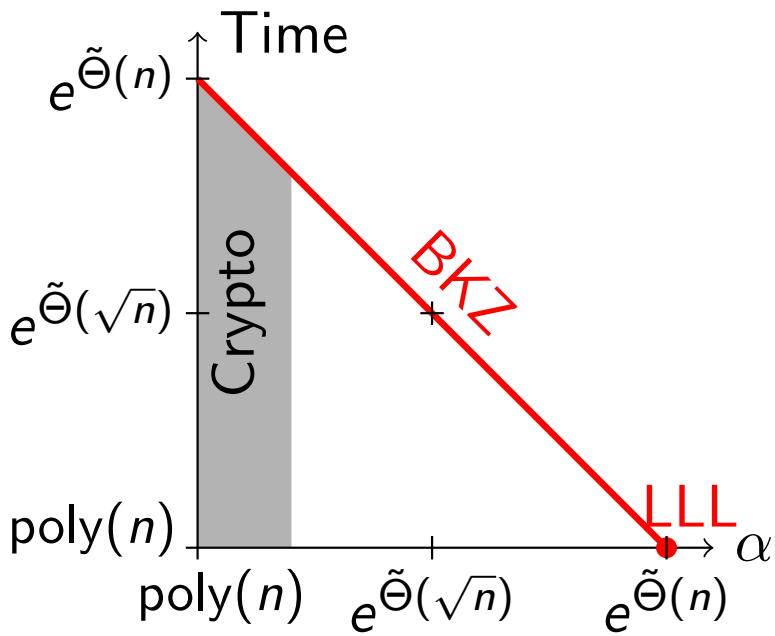
Cycle-count:

	BCNS	**Ours (C ref)**	**Ours (AVX2)**
KeyGen (server)	$\approx 2\,477\,958$	243 748 (243 561)	105 068 (104 874)
Key gen + shared key (client)	$\approx 3\,995\,977$	349 492 (349 686)	141 484 (141 656)
Shared key (server)	$\approx 481\,937$	82 076	29 816

Outline

Best known attack on general lattices

Theoretically: α denotes the approximation factor



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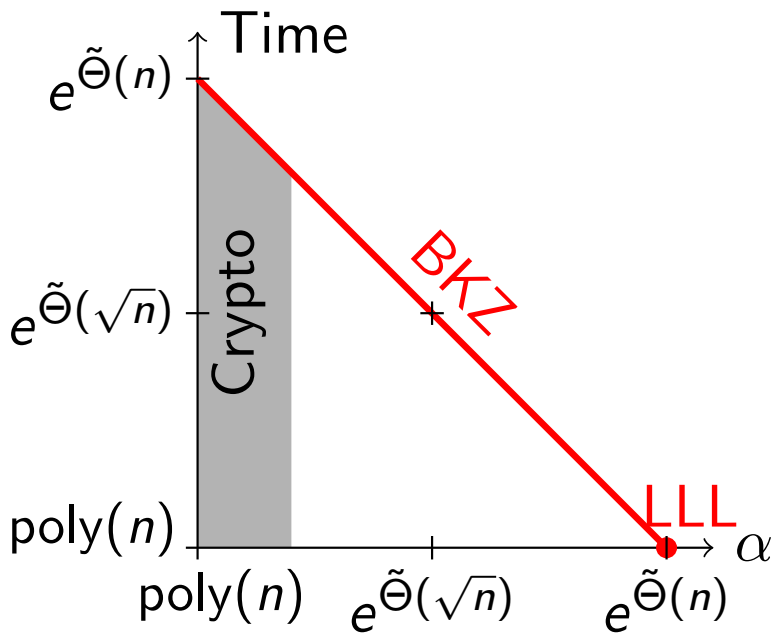

Concretely:

- Too little research compared to theoretical constructions
- Claims hard to reproduce (source code or data missing)
- Many avenue for improvements

Algebraic weaknesses ?

Several recent approaches:

- Evaluation attacks [ELOS15]
  Debunked: improper noise distribution [CIV16, Pei16]
- Overstretched NTRU with subfields [ABD16]
  Debunked: no ring nor subfield needed [KF16]
- Quantum SG-PIP algorithm [CGS14, BS16, CDPR16]
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Quantum PIP and Short Generator recovery

A series of work has shown the following:

SG-PIP in Quantum Poly-time [CGS14, BS16, CDPR16]

Given a **principal ideal** $\mathfrak{I}$ recovering a **short** generator g s.t. $g\mathcal{R} = \mathfrak{I}$, can be done in classical poly-time for

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Short Stickelberger Class Relations & Application to Ideal-SVP [Cramer, D., Wesolowski, 2016]

Our result

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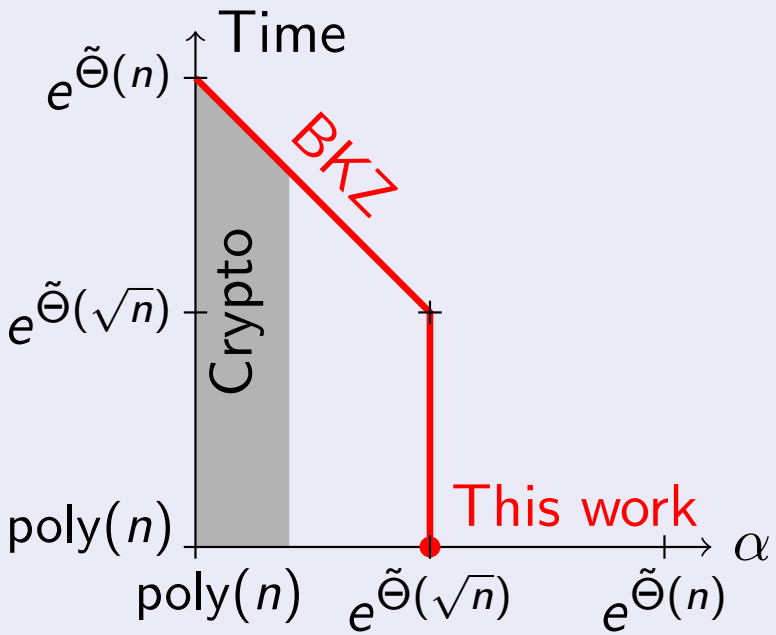

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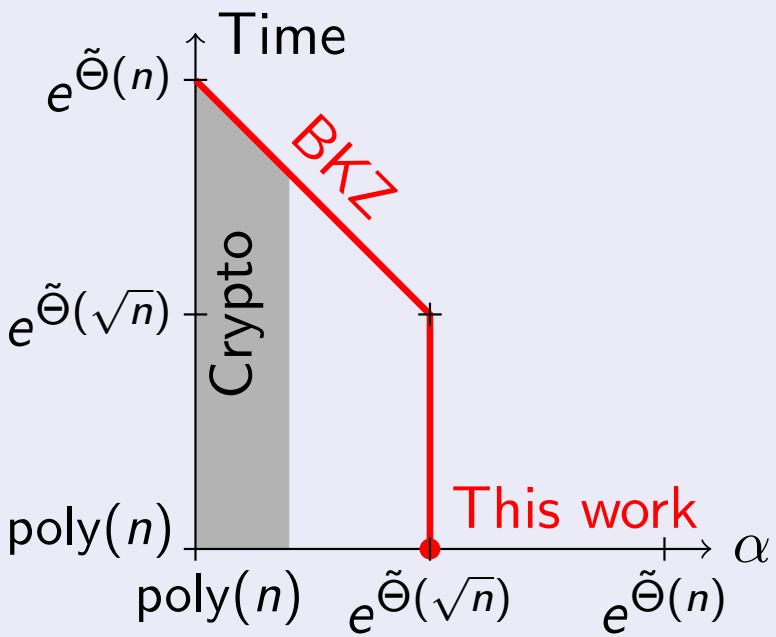

Impact and limitations

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- **Hardness gap** between SVP and Ideal-SVP
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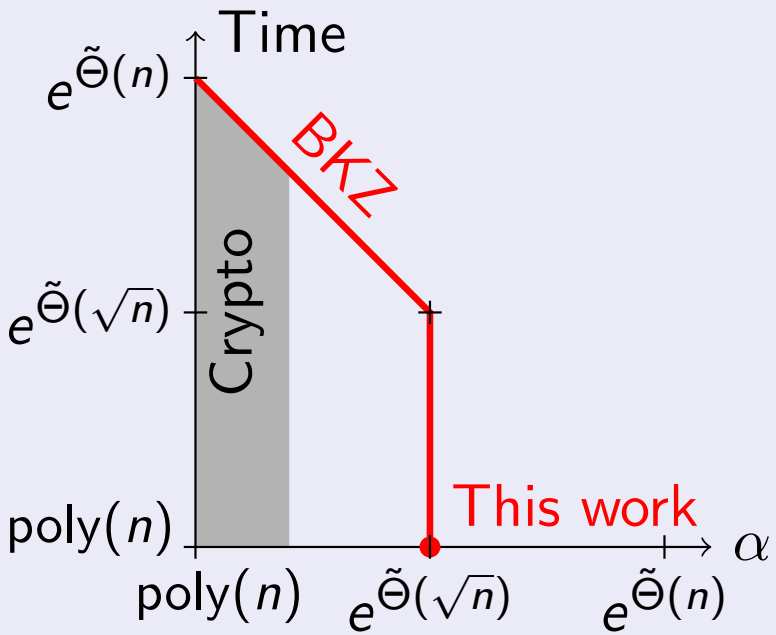

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$\Rightarrow$ start favoring **weaker assumptions** ?

Should we worry ?

Encryption/Key-Exchange/Signature

I don't: the approximation needed seems way too small to be reached by this approach.

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Advanced Scheme (FHE, ABE, iO)

I do: most proposal way too aggressive, both in theory and practice (ignoring dual, sparse secret, ad-hoc assumptions . . .)

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Advanced Scheme (FHE, ABE, iO)

I do: most proposal way too aggressive, both in theory and practice (ignoring dual, sparse secret, ad-hoc assumptions . . .)

Should you trust my gut feeling? **NO !**

- Be as conservative as you can afford to be
- Be patient, wait for the test of time avoid the MLM fiasco

Outline

Frodo [BCD+16]: Take off the ring (A fallback scheme)

Similar design principal applied without ring structure:
(Joint work with NXP, Google and Microsoft)

- Asymptotically, communication grows by $\tilde{\Theta}(\sqrt{\kappa})$
  Naively this would have been $\tilde{\Theta}(\kappa)$
- Can be a bit less conservative (less worries on new attacks)
- No NTT restriction on the choice of q and n
  $q = 2^k$ for efficiency, fine grained choice of n

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Scheme	n	q	**dist.**	bit exchanged	**failure**	**bandwidth**
Challenge	352	2^{11}	D_1	$1 \cdot 8^2 = 64$	$2^{-41.8}$	7.75 KB
Classical	592	2^{12}	D_2	$2 \cdot 8^2 = 128$	$2^{-37.2}$	14.22 KB
Recommended	752	2^{15}	D_3	$4 \cdot 8^2 = 256$	$2^{-36.5}$	22.57 KB
Paranoid	864	2^{15}	D_4	$4 \cdot 8^2 = 256$	$2^{-35.8}$	25.93 KB

Frodo: Security and performances

Security:

Scheme	Attack	**Post-reduction** C	Q	P
Classical	Primal	**132**	120	95
	Dual	**130**	119	94
Recomm.	Primal	157	**143**	113
	Dual	156	**142**	112
Paranoid	Primal	192	175	**139**
	Dual	191	174	**138**

Performances :

TLS.ECDSA+Frodo only 1.6 slower than TLS.ECDSA

Outline

Other alternatives to Powers-of-two Cyclotomic Ring-LWE

Very little diversity so far, most practical scheme use:

$$\mathcal{R} = \mathbb{Z}[\zeta_{2^k}] = Z[X]/(X^{2^{k-1}} + 1))$$

- Theory exists [LPR13] for any cyclotomic ring $\mathcal{R} = \mathbb{Z}[\zeta_m]$
  with complications [DD12], sometime ignored in practice (Helib)
- Should extend to $\mathbb{Z}[\omega_p + \bar{\omega}_p]$, p a safe prime
  Related to cyclotomic, but has no subfields [ABD16]
- Other rings suggested [BCLvV16]: $\mathcal{R} = \mathbb{Z}[X]/(X^p + X + 1)$
  No automorphisms (so no security proof for decisional $\mathcal{R}$-LWE)
- Module-LWE [LS15]: a trade-off between Ring-LWE and LWE
  At least as hard as Ring-LWE, but less efficient.

A concrete Module-LWE based scheme (WIP)

Work in progress:

- $\mathcal{R} = \mathbb{Z}[\omega_{512}]$ module rank $r = 3$, $q = 7681$
  Communication $\approx$ 2.7KBytes, Security > 128 PQ-bits
- $\mathcal{R} = \mathbb{Z}[\omega_p + \bar{\omega}_p]$??
- $\mathcal{R} = \mathbb{Z}[X^p + X + 1]$, ??

Compared to NewHope:

Less structure $\Rightarrow$ more confidence, yet, BETTER performances !

Outline

An implicit error correcting code

LWE-Encrypt :

$$\mathbf{c} = \mathbf{A}\mathbf{s} + \mathbf{e} + \lfloor \frac{q}{2} \rfloor \mathbf{m}$$

LWE-Decrypt :

$$\mathbf{m} = \left\lfloor \frac{2}{q}(\mathbf{c} - \mathbf{A}\mathbf{s}) \right\rceil$$

Implicit encoding-decoding

$$\{0, 1\} \to \mathbb{Z}/q\mathbb{Z},$$

applied component-wise.

Don't we know better ?!

Binary codes

Add an outer binary ECC

+ Asymptotically efficient codes of maximal capacity (e.g. Turbo codes)
+ Very widespread
+ Well known by many cryptographers
– Beware of lack of independance between errors
? Resistance to timing/cache attacks

Lattice-codes

Equivalent of ECC for iscrete data over continuous channel

+ Interact optimally with lattice based crypto,
  no information wasted
+ Less understood, especially by cryptographers
− no good asymptotic construction
? Resistance to timing/cache attacks ?

Two Lattice-code tentatives

In NewHope:

- Root-lattice D_4 to encode 1 bit into 4 coordinates
  (256 bits key agreed, for a dimension $n = 1024$)
- No performance gain because of parameter constraints
  can only gain security, which was already overshot

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Cryptographic decoding of the Leech Lattice [vanPoppelen, MSc]:

- Revisiting the best Leech Lattice Decoder
- Cache-attack resistant implementation ($\approx$ 100 000 cycles)
- Estimated bandwidth gain on a Frodo-like scheme 10 to 20%

Still a lot of nice theory and practice work to do on this topic !

Thanks

Questions ?


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